Homework 2

(Pen and paper)
(Ask the teacher for the values of the parameters *a* and *b*)

Question 1. (chapter 9. 3)
Consider the following steady, two-dimensional, incompressible velocity field
$\vec{V} = a\vec{i} - e^{-bx}\vec{j}$
(time unit is 1s, length unit is 1m)
Find an expression for the stream function ψ and plot the stream lines for
$\psi = 0, 1, 2, 3, 4, 5.$

Question 2. (chapter 9. 1)

a) Prove the following formulas
a1) $\nabla(fg) = f\nabla g + g\nabla f$
a2) $\nabla \bullet (f\vec{F}) = (\nabla f) \bullet \vec{F} + f(\nabla \bullet \vec{F})$
where $f = f(x, y, z)$ and $g = g(x, y, z)$ are scalar functions
$\vec{F} = (P(x, y, z), Q(x, y, z), R(x, y, z))$ is a vector function (vector field)
and $\nabla = (\vec{i}\,\frac{\partial}{\partial x}, \vec{j}\,\frac{\partial}{\partial y}, \vec{k}\,\frac{\partial}{\partial z})$ is "del" or "nabla" operator.

Question 3. (chapter 9. 1)

Expand (write without using del ∇ or divergence operator) the following two forms of the continuity equation and show that they are equivalent:
eq1: $\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho\vec{V}) = 0$
eq2: $\frac{D\rho}{Dt} + \rho(\nabla \bullet \vec{V}) = 0$

Question 4. (chapter 4)
Consider the following velocity field
$\vec{V} = (x + z)\vec{i} + (ay + z)\vec{j} + bz\vec{k}$
(time unit is 1s, length unit is 1 dm)

i) Calculate
a) the linear strain rates b) the volumetric strain rate
c) the strain rate tensor
ii) Determine if this flow is incompressible.
iii) Calculate the angular velocity vector and find if the flow is rotational. If so determine if fluid particles rotate clockwise or counterclockwise.
iv) Find the path line for the particle that starts in the point P(0,0,3) for t= 0s.
Determine the location of this particle at t=1 s