Question 1.
Consider the following steady, two-dimensional, incompressible velocity field
$\vec{V} = 2\vec{i} - e^{-x}\vec{j}$

a) Find an expression for the stream function ψ

b) Plot the stream lines for $\psi = 0, 1, 2, 3.$

Solution:

a) For a steady two-dimensional, incompressible velocity field $\vec{V} = (u, v)$ the stream function ψ is defined by the following equations:

$$\frac{\partial \psi}{\partial y} = u(x, y), \qquad \text{(eq1)}$$

$$\frac{\partial \psi}{\partial x} = -v(x, y). \qquad \text{(eq2)}$$

The first equation gives

$$\frac{\partial \psi}{\partial y} = 2 \Rightarrow \psi(x, y) = 2y + f(x).$$

We substitute this in the (eq2) and get

$$f'(x) = e^{-x} \Rightarrow f(x) = -e^{-x} + C.$$

Thus $\psi(x, y) = 2y - e^{-x} + C$.

We can take C=0 and so $\psi(x, y) = 2y - e^{-x}$.

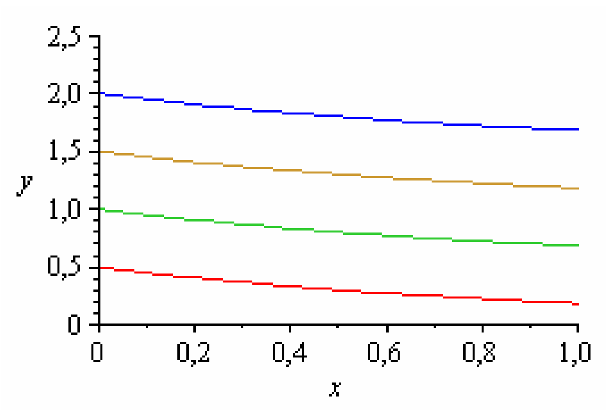
Answer a: $\psi(x, y) = 2y - e^{-x}$

b) The stream function $\psi = 2y - e^{-x}$ is constant on a streamline.

From $\psi = 2y - e^{-x}$ we get $y = \dfrac{\psi + e^{-x}}{2}$.

Now we substitute $\psi = 0, 1, 2, 3$ and get the following functions:

$$y_0 := \frac{1}{2} e^{-x} \qquad y_1 := \frac{1}{2} e^{-x} + \frac{1}{2} \qquad y_2 := \frac{1}{2} e^{-x} + 1 \qquad y_3 := \frac{1}{2} e^{-x} + \frac{3}{2}$$



Question 2.
Find an expression for the stream function ψ for the following steady, two-dimensional, incompressible velocity fields

a) $\vec{V} = (x + y)\vec{i} - y\vec{j}$ b) $\vec{V} = 2x\vec{i} + (1 - 2y)\vec{j}$ c) $\vec{V} = 3\vec{i} + (1 + e^{-2x})\vec{j}$

Answer:

a) $x\,y + \frac{1}{2} y^2$ **b)** $2\,x\,y - x$ **c)** $3\,y - x + \frac{1}{2} e^{-2\,x}$