Example1 (***Pathline*****):**

Consider the following velocity field

$\vec{V} = 8x\vec{i} + (2y + x)\vec{j} + (x + z)\vec{k}$.

Find the pathline for the particle that starts at the point P(1,2,3) for t= 0.

Determine the location of this particle at t=1.

Solution:

To find the pathline that starts at the point P we solve the system:

$$eq1 := \frac{d}{dt} x(t) = 8\,x(t)$$

$$eq2 := \frac{d}{dt} y(t) = 2\,y(t) + x(t)$$

$$eq3 := \frac{d}{dt} z(t) = x(t) + z(t)$$

$$x(0) := 1, \quad y(0) := 2, \quad z(0) := 3$$

Since the system is linear with constant coefficients we can, for instance, use Laplace transforms to solve it.

Transforming each equation we obtain the following three algebraic equations:

$$eq1 := s\,X - 1 = 8\,X$$

$$eq2 := s\,Y - 2 = 2\,Y + X$$

$$eq3 := s\,Z - 3 = X + Z$$

From this system we get

$$X := \frac{1}{s - 8}$$

$$Y := \frac{2\,s - 15}{s^2 - 10\,s + 16}$$

$$Z := \frac{3\,s - 23}{s^2 - 9\,s + 8}$$

Now by partial fractions

$$X := \frac{1}{s - 8}$$

$$Y := \frac{1}{6\,(s - 8)} + \frac{11}{6\,(s - 2)}$$

$$Z := \frac{1}{7\,(s - 8)} + \frac{20}{7\,(s - 1)}$$

Hence, by using the inverse Laplace transform, we get the equations for the pathline that starts at the point P(1,2,3) for t= 0:

$$x := e^{8\,t}$$

$$y := \frac{1}{6}\,e^{8\,t} + \frac{11}{6}\,e^{2\,t}$$

$$z := \frac{1}{7}\,e^{8\,t} + \frac{20}{7}\,e^{t}$$

At t=1 the particle is at the point

Q=(x(1),y(1),z(1))=(2981, 510, 434).