Example 1 (Pathline):
Consider the following velocity field
\[ \mathbf{V} = 8x\mathbf{i} + (2y + x)\mathbf{j} + (x + z)\mathbf{k}. \]
Find the pathline for the particle that starts at the point P(1,2,3) for \( t = 0 \). Determine the location of this particle at \( t = 1 \).

Solution:
To find the pathline that starts at the point P we solve the system:

\[
\begin{align*}
\frac{dx}{dt} &= 8x(t) \\
\frac{dy}{dt} &= 2y(t) + x(t) \\
\frac{dz}{dt} &= x(t) + z(t)
\end{align*}
\]

Since the system is linear with constant coefficients we can, for instance, use Laplace transforms to solve it.
Transforming each equation we obtain the following three algebraic equations:

\[
\begin{align*}
eq 1 &:= sX - 1 = 8X \\
eq 2 &:= sY - 2 = 2Y + X \\
eq 3 &:= sZ - 3 = X + Z
\end{align*}
\]

From this system we get

\[
\begin{align*}
X &= \frac{1}{s - 8} \\
Y &= \frac{2s - 15}{s^2 - 10s + 16} \\
Z &= \frac{3s - 23}{s^2 - 9s + 8}
\end{align*}
\]

Now by partial fractions

\[
\begin{align*}
X &= \frac{1}{s - 8} \\
Y &= \frac{1}{6}(s - 8) + \frac{11}{6}(s - 2) \\
Z &= \frac{1}{7}(s - 8) + \frac{20}{7}(s - 1)
\end{align*}
\]

Hence, by using the inverse Laplace transform, we get the equations for the pathline that starts at the point P(1,2,3) for \( t = 0 \):

\[
\begin{align*}
x &= e^{8t} \\
y &= \frac{1}{6}e^{8t} + \frac{11}{6}e^{2t} \\
z &= \frac{1}{7}e^{8t} + \frac{20}{7}e^{t}
\end{align*}
\]

At \( t = 1 \) the particle is at the point
\( Q = (x(1), y(1), z(1)) = (2981, 510, 434) \).